Chapter 8 Probability Part II Multiplication Rules

I. Special rule of multiplication

A. Events A and B are **independent** when event A happening does not affect the probability of event B happening.

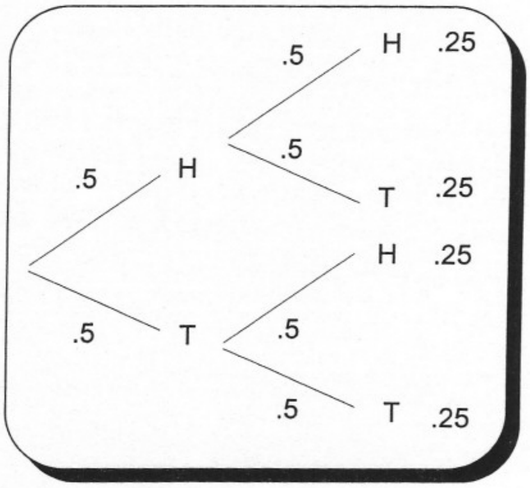
B. The intersection of two events represents how often they happen together.

1. This probability of this intersection is called **joint probability.**
2. When two events are independent, their joint probability is the product of their individual probabilities.

$$P(A \text{ and } B) = P(A) \times P(B)$$

C. Flipping a coin results in independent events because the outcome of the first flip (event #1) does not affect the outcome of the second flip (event #2).

$$P(H \text{ and } H) = P(H) \times P(H) = (.5)(.5) = .25$$



D. Special multiplication problems can also be solved with a **contingency table** and a **tree diagram**.

Toss 1 / Toss 2	P(H) = .5	P(T) = .5	Totals
P(H) = .5	.25	.25	0.50
P(T) = .5	. 25	.25	0.50
Totals	.50	.50	1.00

1. The probability of a third head is still .5 and the probability of three heads in a row is (.5)(.5)(.5) = .125.
2. The P(A) is referred to as **marginal probability** because its probability is located in the margins of a contingency table.
3. For independent events, joint probability P(A and B) is the product of marginal probability (see the highlighted boxes of the contingency table).

II. General rule of multiplication

A. Events A and B are **dependent** when event A happening has an affect on the probability of event B happening.

B. Rather than simply multiplying $P(A) \times P(B)$, the P(B) is adjusted for the effect of A having happened.

C. The idea of event A happening first and affecting B is known as **conditional probability**. A conditional probability statement would be written P(B I A) and read the probability of B given A.

D. When events are dependent, A affects B and the general rule of multiplication is appropriate.

$$P(A \text{ and } B) = P(A) \times P(B \mid A)$$

Note that P(B I A) is weighted by the probability of A happening.

E. Suppose Linda wants to determine the probability of advertising expenditures being greater than $5,000 and sales revenue being greater than $50,000.

P(A > $5,000 and S > $50,000)

= P (A > $5,000) P(S > $50,000 I A > $5,000)

$$= \frac{5}{10} \times \frac{4}{5} = \frac{20}{50} = .4 = 40\%$$

Note: This answer can be read directly from this table. $\frac{4}{10} = .4$

Monthly Advertising and Sales			
Sales Revenue / Advertising Expenditures	Less than or equal to $50,000 (≤ 50)	Greater than $50,000 (> 50)	Totals
Less than or equal to $5,000 (≤ 5)	4	1	5
Greater than $5,000 (> 5)	1	4	5
Totals	5	5	10

Note: The general probability rules for addition and multiplication work for all cases.
The special rule for addition may be used when events are mutually exclusive.
The special rule for multiplication may be used when events are independent.

III. Bayes' theorem

A. Bayes' theorem is used to find the probability of conditional events.

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)} = \frac{P(A) \times P(B|A)}{P(A) \times P(B|A) + P(\tilde{A}) \times P(B|\tilde{A})}$$

B. Logic of Bayes' theorem

1. The condition is that B has occurred. The denominator contains the situations when this happens (the whole).
   a. Therefore, it contains B happening with A plus B happening with $\tilde{A}$.
   b. B happening with A is weighted by how often A occurs.
   c. B happening with $\tilde{A}$ is weighted by how often $\tilde{A}$ occurs.
2. The numerator is the part of the denominator that is of concern. In this case it is *A* happening with B.

C. Linda wants to determine the probability of sales being over \$50,000 when she spends over \$5,000 on advertising. First Bayes' theorem is written using symbols more representative of the problem. Second, substitute and solve.

$$P(>\$50 \mid >\$5) = \frac{P(>50 \text{ and } >5)}{P(>5)}$$

$$= \frac{P(>50) \times P(>5|>50)}{P(>50) \times P(>5|>50) + P(>\tilde{50}) \times P(>5 \mid >\tilde{50})} = \frac{\frac{5}{10} \times \frac{4}{5}}{\frac{5}{10} \times \frac{4}{5} + \frac{5}{10} \times \frac{1}{5}} = \frac{\frac{20}{50}}{\frac{20}{50} + \frac{5}{50}} = \frac{\frac{20}{50}}{\frac{25}{50}} = .8 = 80\%$$

IV. Joint and conditional probability may easily be read from a contingency table converted to decimals.

Monthly Advertising and Sales				
Sales / **Advertising**	Less than or equal to \$50,000	Greater than \$50,000	Totals	The page 46 answer to sales over \$50,000 and advertising over \$5,000 of 40% can be read directly from this chart.
Less than or equal to \$5,000	0.40	0.10	0.50	The answer to the conditional statement above can be read off the chart as .4 divided by .5 or 80%.
Greater than \$5,000	0.10	0.40	0.50	Advertising and sales are dependent so the special rule for multiplication does not apply. Note that joint probability is not the product of marginal probability.
Totals	0.50	0.50	1.00	

V. Counting relevant outcomes

A. As problems become more complex, counting total outcomes and outcomes of interest will also be more complex.

B. **The counting rule:** If one event can happen M ways and a second event can happen N ways, then the two events can happen in sequence (M)(N) ways. Linda wants to visit her 3 competitors, each of whom have 2 stores. There are (3)(2) = 6 stores she can visit. The total counting for three events would be (M)(N)(O).

C. **The factorial rule** involves arranging N available items.

1. Linda can visit the 6 stores of her competitors using 6! alternative routes.
2. $N! = 6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$ alternative routes
3. When she begins, she has 6 alternatives. Having been to a store, she then has 5 alternatives, then 4, etc.

D. **The permutation rule** involves arranging R of N available items.

1. Order is important as a, b, c and c, a, b are different and each is counted as an outcome.
2. Here is how many ways Linda could arrange 4 of 7 posters as a window display. N = 7 and R = 4

$$_NP_R = \frac{N!}{(N-R)!} = \frac{\text{Totality}}{\text{What is not of interest}} = {}_7P_4 = \frac{7!}{(7-4)!} = \frac{7\times6\times5\times4\times3\times2\times1}{3\times2\times1} = 7\times6\times5\times4 = 840$$

E. **The combination rule involves** choosing (not arranging) R of N available items. Because items are not being arranged, order is not important. Items abc and cba are the same and are not counted twice.

1. Just hanging (not arranging) 4 of 7 posters has fewer possibilities because order doesn't count.

$$_NC_R = \frac{N!}{(N-R)!(R!)}$$

$$_7C_4 = \frac{7!}{(7-4)!4!} = \frac{7\times6\times5\times4\times3\times2\times1}{3\times2\times1\times4\times3\times2\times1} = \frac{7\times6\times5}{3\times2\times1} = 35$$

2. The use of R! in the denominator eliminates the multiple counting of items of interest.